Differentiating e^x

1.

Differentiate the following with respect to x.

a e^{6x} **b** $e^{-\frac{1}{3}x}$ **c** $7e^{2x}$

d $5e^{0.4x}$ **e** $e^{3x} + 2e^x$ **f** $e^x(e^x + 1)$

Hint For part **f**, start by expanding the bracket.

2.

Find the gradient of the curve with equation $y = e^{3x}$ at the point where

a $x = 2$ **b** $x = 0$ **c** $x = -0.5$

3.

The function f is defined as $f(x) = e^{0.2x}$, $x \in \mathbb{R}$. Show that the tangent to the curve at the point $(5, e)$ goes through the origin.

4.

a Write down the gradient of the graph of

i $y = e^x$ at the point $(2, e^2)$

ii $y = e^{-x}$ at the point $(2, e^{-2})$

b Find the equation of the tangent to the curves at the given points.

5.

Find the point where the tangent to the curve $y = e^{\frac{1}{2}x}$ at the point $(4, e^2)$ intersects the straight line $x = 6$. Show your working.

6.

Find the x-intercept of the normal to the curve $y = e^{2x}$ at the point where $x = 1$

7.

Differentiate with respect to t

a $7 - 2e^t$

c $e^t + t^5$

d $t^{\frac{3}{2}} + 2e^t$

h $7t^2 - 2t + 4e^t$

8.

Find the value of $f'(x)$ at the value of x indicated in each case.

a $f(x) = 3x + e^x$, $\quad x = 0$

d $f(x) = 5e^x + \dfrac{1}{x^2}$, $\quad x = -\frac{1}{2}$

9.

Find the coordinates and the nature of any stationary points on each of the following curves.

a $y = e^x - 2x$

d $y = 4x - 5e^x$

10.

Given that $y = x + ke^x$, where k is a constant, show that

$$(1 - x)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0.$$

11.

a Find an equation for the normal to the curve $y = \frac{2}{5}x + \frac{1}{10}e^x$ at the point on the curve where $x = 0$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

b Find the coordinates of the point where this normal crosses the x-axis.

12.

a Sketch the curve with equation $y = e^x + k$, where k is a positive constant.

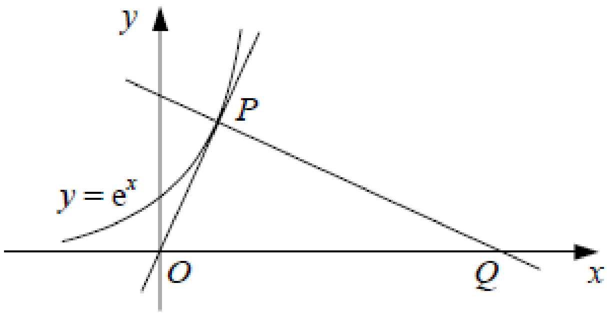
Show on your sketch the coordinates of any points of intersection with the coordinate axes and the equations of any asymptotes.

b Find an equation for the tangent to the curve at the point on the curve where $x = 2$.

Given that the tangent passes through the x-axis at the point $(-1, 0)$,

c show that $k = 2e^2$.

13.



The diagram shows the curve with equation $y = e^x$ which passes through the point P (p, e^p). Given that the tangent to the curve at P passes through the origin and that the normal to the curve at P meets the x-axis at Q,

a show that $p = 1$,

b show that the area of triangle OPQ, where O is the origin, is $\frac{1}{2}e(1 + e^2)$.

14.

The growth of a population of mice is modelled by $N = 50e^{0.1t}$, where N is the number of mice and t is measured in weeks.

(i) After how many weeks is the number of mice greater than 200?

(ii) What is the rate of increase in the population after 5 weeks?

(iii) Show that $\frac{dN}{dt} = kN$, giving the value of k.

(iv) What is the rate of increase in the population when there are 200 mice?

(v) Explain why this model is unlikely to be appropriate as N becomes very large.